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GRAPHICAL ANALYSIS OF THE SENSITIVITIES
OF ATCAL IN THE FORCEM MODEL

by

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GRAPHICAL ANALYSIS OF THE SENSITIVITIES OF ATCAL IN THE FORCEM MODEL

by

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requirements for the degree of

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ABSTRACT

This is an analysis of an attrition process in the context of its theater model. A graphical data analysis of the sensitivities of ATCAL with respect to FORCEM operational data was performed. Given the ATCAL results from various FORCEM runs, the sensitivity of ATCAL within the FORCEM model to the effects of frontage of engagement and presence of important weapon systems was investigated.

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I. INTRODUCTION

A. INTRODUCTION

Verification and validation of computer simulations of large complex systems such as theater level combat operations is difficult at best. In practice, this is not a single event, but continues to occur throughout the life of the models involved. While the basic intent is to insure that the model is true to its design and that the design reasonably approximates reality, the effort also yields insight into model sensitivity and possible utility of information generated by the model.

The purpose of this project is to analyze an attrition process in the context of its theater model. This study will examine sensitivity with respect to certain variables, to verify that the process, in this context, provides an intuitively appealing output, and to look for items which may be used as simple performance predictors in other aspects of the model. The process under investigation is the Attrition Calibration (ATCAL) process, used by the Army Concepts Analysis Agency (CAA) as the attrition mechanism in its theater-level models, Concepts Evaluation Model (CEM) and Force Evaluation Model (FORCEM).

B. BACKGROUND

CEM is a deterministic theater model of ground and air combat which has been developed through several versions since about 1970. [Ref. 1:pp. 1-11]

FORCEM is a more detailed computer simulation model portraying combat, combat support, and combat service support in a theater of operations. It is a fully-automated, deterministic, and time-stepped model, with a module for each major functional area being activated once each twelve-hour time cycle. It was developed by CAA during the period 1982-1984, and is now operational. Applications are in studies that determine the capabilities of current combat forces; requirements for support forces; and requirements for personnel, supplies, and major items of equipment. [Ref. 2:p. 2]

ATCAL is an Attrition Calibration Model which was developed in the early 1980's by CAA for use in the CEM and FORCEM theater level simulations. ATCAL consists of a number of non-linear equations which can be used to compute weapon systems attrition of opposing forces (if values for several input parameters are known). The same equations can be used "backwards" to determine values for the parameters from the output of a higher resolution model such as CAA's divisional Combat Sample Generator (COSAGE). The ATCAL process contains both an attrition model and a consistent calibration procedure.

The ATCAL model does not step through time, but rather reflects the casualty rates of the division level engagement to which it is calibrated. [Ref. 3:p. 6-1]

Operational use of FORCEM in 1987 and early 1988 generated a backlog of corrections, enhancements, and new developments which were desired in the model. In May 1988, a task force was constituted within CAA to address these needs in a two-phased program. As part of phase one, an analysis was conducted to determine how certain aspects of the model, including the mechanics of forming combat engagements and certain computer-science related issues, affected the attrition process.

Other tests have also been conducted at CAA which examined sensitivities of various aspects of the COSAGE-ATCAL-FORCEM integrated system of models. These controlled tests have generally shown ATCAL to be robust, but have pointed out some areas of interest for further testing. These areas and the areas of interest found through the task force phase one analysis form the base from which this study is derived. The specific focus of this study is to examine some of the sensitivities of ATCAL in its operational setting. [Ref. 4]

ATCAL itself is sensitive to rate of fire and engagement frontage [Ref. 3:pp. 6-11]. The actual effect of these sensitivities has never been analyzed in FORCEM operational data. It is appropriate to analyze model results with respect to these sensitivities to verify that the results are sensible

in the simulation. Where these parameters can be adequately controlled, they may also provide mechanisms through which other desired effects can be infused into the FORCEM Model [Ref. 5:p. 1].

C. PURPOSE OF RESEARCH EFFORT

The purpose of this study is to perform a graphical data analysis of the sensitivities of ATCAL with respect to FORCEM operational data. Given a history of ATCAL results from various FORCEM runs, appropriate graphical techniques will be applied to illustrate the sensitivity of ATCAL within the FORCEM Model to the effects of frontage of engagement and presence of important weapon systems.

This is important in light of the previous tests done on sensitivities of the COSAGE-ATCAL-FORCEM integrated system of models. Graphical analysis will provide the first step in the effort to understand the sensitivity of ATCAL within the FORCEM model.

II. DATA AND METHODOLOGY

A. DATA

The data used in this analysis is drawn from 20 day combat simulations between a Blue defending force and a Red attacking force. This FORCEM basic data will be referred to as FB. A comparative run was also made using a version of FORCEM which restricted the number of engagements each unit could be involved in per time period so that the number of extremely small engagements was reduced. This set of data of FORCEM restricted engagements will be referred to as FR.

A third set of data was developed by incorporating the version of ATCAL found in CEM into the FORCEM model. This was done to examine the effect in FORCEM of certain processing differences found in the CEM ATCAL. This data set will be referred to as FC.

The characteristics of each of the data sets can be seen in Tables I and II. The data sets were run through the Statistical Analysis System (SAS) using the procedure Univariate to get the results for Table I. The complete output for each of the data sets is in Appendix A.

TABLE I

DATA SETS

FB - FORCEM BASIC DATA
FR - FORCEM RESTRICTED ENGAGEMENTS DATA
FC - FORCEM DATA USING CEM's VERSION OF ATCAL

DATA SET	NUMBER OF OBSERVATIONS	NUMBER OF VARIABLES
FB	47487	18
FR	19985	17
FC	652*	15

* ENGAGEMENT-LEVEL TOTALS ONLY

TABLE II

VARIABLES

TIME	Day or night engagement
BRD	Posture of the blue forces (i.e., defend, delay)
BLUE	Blue unit involved in the engagement
FBLUE	Fraction of the blue unit involved in the engagement (i.e., .33, .5)
RED	Red unit involved in the engagement
FRED	Fraction of the red unit involved in the engagement (i.e., .65, .10)
FRONT	How large a front size the engagement takes place on
AST	A number which represents the different assets of both the red and blue units (i.e., 5-tank for blue, 53-tank for red)
OHAND	The number of a particular asset which is available at the beginning of the engagement.
HITS	The number of that particular asset which were hit during the engagement
KKILL	The number of that particular asset which were killed during the engagement
GLOBE	The system value which is assigned that particular asset
TOASCW	The total combat worth of that asset (number of assets on-hand at the beginning of the engagement times their system value)

TABLE II (CONT)

LOASCW	The lost combat worth of that asset at the end of the engagement (the number of assets hit or killed times their system value)
TOBLCW	The total blue combat worth (all the blue assets time their system value sum together at the beginning of the engagement)
LOBLCW	The number of blue assets lost during the engagement times their system value.
INITRA	Initial force ratio (red's total combat worth/blue's total combat worth at the beginning of the engagement)
FINARA	Final force ratio (red's total combat worth at the end of the engagement/blue's total combat worth at the end of the engagement
BTNKO	Number of red tanks on-hand at the beginning of the engagement
RTNKH	Number of red tanks hit during the engagement
RTNKK	Number of red tanks killed during the engagement
BTNKO	Number of blue tanks on-hand at the beginning of the engagement
BTNKH	Number of blue tanks hit during the engagement
BTNKK	Number of blue tanks killed during the engagement

B. METHODOLOGY

Exploratory data analysis techniques are to be utilized to analyze the data described above. The data will be manipulated first using SAS. Subsets of each of the data sets will be put into the GRAFSTAT Package to take advantage of the exploratory data analysis techniques which are available within this package.

Using the techniques provided by both SAS and GRAFSTAT this study will attempt to answer the following questions:

- What effects do frontage changes have on the response of ATCAL in FORCEM?

- Is an increase in final force ratio associated with an increase in frontage?

- How does the presence or absence of important weapon systems in an engagement change the performance of ATCAL in FORCEM results?

- Is there a noticeable change in results when a weapon is absent from an ATCAL engagement?

In reality, an increase in the frontage size might be expected to show an increase in the final force ratio if everything else in the engagement is held constant. We will investigate the response of ATCAL to changing frontage in this study. Similarly, if a weapon system is really important to a unit the absence of this system should cause noticeable changes in the results of the engagements. The killing potential of the unit should be less, so then there are fewer enemy assets being killed. We will attempt to determine if the results from ATCAL confirm this.

There will also be an attempt to identify questions which cannot be answered within the scope of this study or the data provided for future studies.

III. ANALYSIS OF FRONTAGE EFFECTS

Frontage is the width of the area that a defending unit must control. In the military sense, a unit can control the area by physically being there, or by having the ability to observe the area and deliver effective fire onto it.

Given a fixed size for a defending unit (i.e., a fixed combat worth), it would seem that, as the unit is assigned a larger frontage to defend, its ability to accomplish the mission would decrease as the size of the front increased. Alternatively, a fixed size unit attacking on decreasing frontages might expect increased losses as its maneuver room is reduced. Frontage changes would naturally be a part of any combat model which has attacking and defending forces.

In FORCEM, the frontage associated with a specific engagement is calculated as follows: Engagement Frontage = 1/2 [Fraction of Red Division in that Engagement * Red Division Frontage + Fraction of Blue Division in that Engagement * Blue Division Frontage]. The division frontages are computed from corps frontages as 1/N * [Corps Frontage] where N is the number of divisions in the corps. Corps frontage is determined from the corps boundaries which are input by the model operator and intended to represent realistic frontages for the units and operations involved.

Units are apportioned for combat against enemy units by a process based on range. [Ref. 2:pp. 6-11]

The data sets from both versions of ATCAL were investigated to see if changes in frontage size had any effect on the final force ratio. Final force ratio is the ratio of the number of operational red weapon systems times their system values, divided by the number of operational blue weapon systems times their system values at the end of the engagement.

An example of how final force ratio is computed: At end of an engagement all the operational red weapon systems are counted, (5 Tanks, 6 BMPs, 4 Helicopters), the number of operational systems is then multiplied by the system value for that system (Tank = 1.0, BMP = .4, Helicopter = .7) and all of these numbers are added together to get the combat worth of the red unit which is also the numerator of the final force ratio. The system value of a system is input by the model operator. The same procedure is done for the blue units and this total combat worth is used as the denominator of the ratio. [Ref. 2]

It is important to take into account the initial force ratio when examining the relationship between frontage and final force ratio since this will also have an effect on final force ratio. The initial force ratio (that is the ratio of the number of red weapon systems times their system values, divided by the number of blue weapon systems times their

system values at the beginning of the engagement), final force ratio, and frontage size were the three variables in the data sets which were investigated.

To investigate the possibility that changes in the frontage size have no effect on the final force ratio a subset of each of the data sets were made. The subset contained all the engagements where the forces were in the most intense conflict during the day.

Figure 1 depicts a graph of the subset for data set FC. It shows the final force ratio grows rather slowly with the initial force ratio for any fixed range of frontage sizes. This is not what is expected to happen, there should be a tendency for the final force ratio to increase as the frontage size increases. Figure 2 displays two graphs. The top graph shows the empirical density of the initial force ratios vs. the different frontage sizes and the lower graph shows the empirical density of the final force ratios vs. the different frontage sizes. In comparing the two graphs it is apparent at all the frontage sizes that the final force ratios have longer tails toward the higher ratios than do the initial force ratios. The largest changes take place between a frontage size of 5500 meters and 6500 meters. Figure 2 does show that there is some change between final and initial force ratio at all frontage sizes. Although the greatest change takes place where the frontage sizes goes from 5500 meters to 6500 meters, this is not what is expected, the largest changes

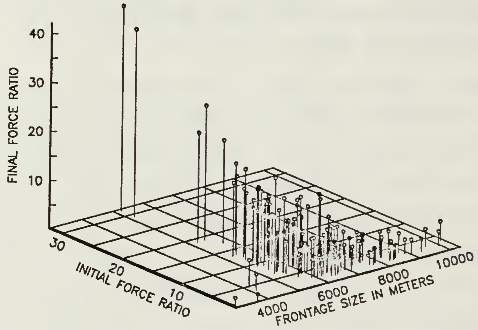
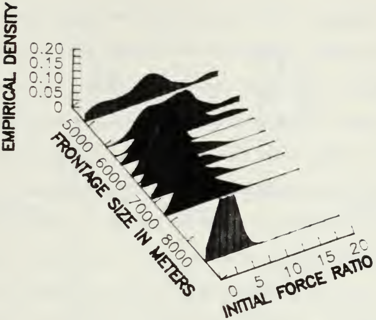


Figure 1. Final Force Ratio Plotted as a Function of Initial Force Ratio and Frontage Size (N=143), Data=FC



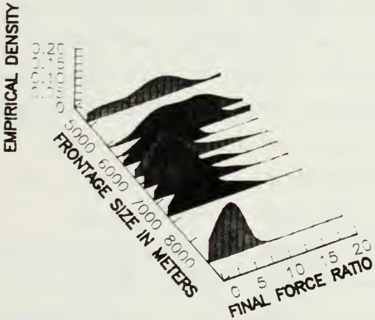


Figure 2. Empirical Densities of Initial and Final Force Ratios Conditioned on Frontage Size (N=143) (Data=FC)

should be seen at the higher frontage sizes. Thus, increases in frontage size do not have a large effect on the final force ratio.

Figure 3 depicts the final force ratio plotted against the initial force ratio conditioned on the frontage size changes. For every point that lies above the line y=x, blue is losing his combat worth at a greater rate than red considering the initial force ratio and the frontage size. Below the line, red is losing at a greater rate. Thus, there is a tendency at the nominal Cosage 3:1 ratio (this is the ratio at which it would be favorable for a unit to attack and win the engagement), red is consistently losing a higher combat worth proportion relative to blue over all different frontage sizes. It is not until the initial force ratio is larger than 4:1 that the blue force consistently loses a higher combat worth proportion relative to red. It would seem appropriate that at high initial force ratios we should expect to see decisive outcomes, but this is not shown true in Figure 3. The growth of the final force ratio as the initial force ratio increases is relatively slow across the entire range.

A graph of the subset of FR data set is Figure 4. As with the FC data set it does not appear that there is any change in the final force ratio as the frontage increases from this view. The graph in Figure 5 represents the empirical densities of both the initial force ratio and final force ratio against different frontage sizes. The densities at all

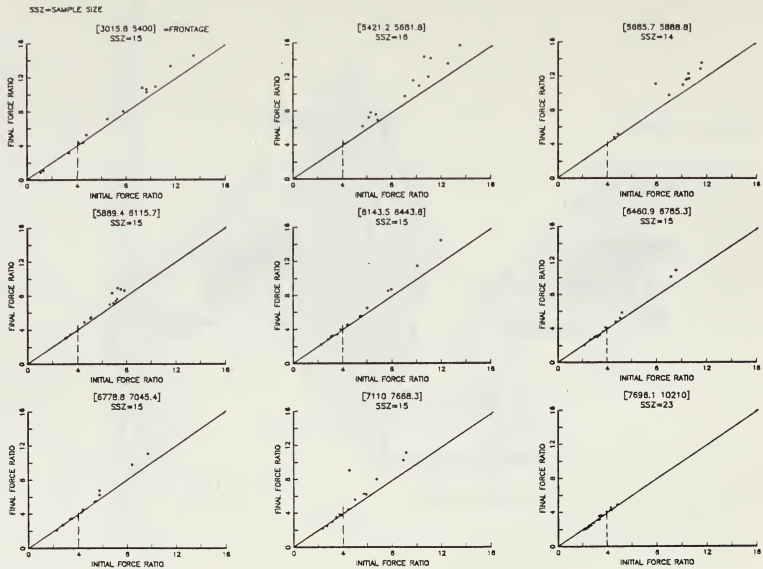


Figure 3. Conditional Scatter-plots of Final vs. Initial Force Ratios Conditioned on Frontage (Data=FC)

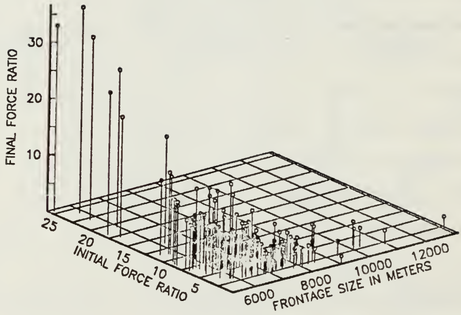
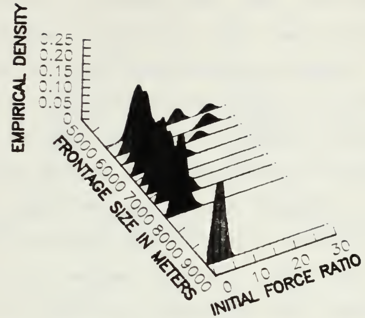


Figure 4. Final Force Ratio Plotted as a Function of Initial Force Ratio and Frontage Size (N=133), Data=FR



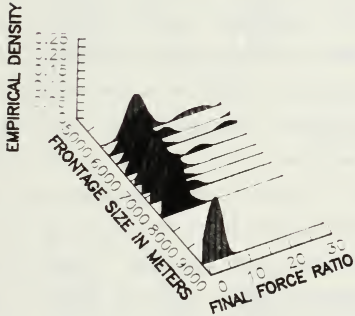


Figure 5. Empirical Densities of Initial and Final Force Ratios Conditioned on Frontage Size (N=133) (Data=FR)

frontage sizes are less peaked in the graph of the final force ratios than in the graph of the initial force ratio. The tails in the final force ratio graph are longer toward the higher frontage sizes than are those in the initial force ratio graph. There is no particular region of frontage sizes where the increase of final force ratio is that much greater than the initial force ratio as shown in Figure 2. There is a change between the distribution of initial and final force ratios at all frontage sizes, but the change between the ratios does not tend to be higher at the high frontage sizes as would be expected. Figure 6 depicts the same graph as Figure 3, except that the subset of FR data set is used in place of the FC data set. Again it is shown that not until the red force has obtained an initial force ratio larger than a 4:1 ratio does the blue force consistently lose a higher combat worth proportion relative to the red force and this is true over the ranges of frontage sizes. There is no evidence of a decisive outcome at high initial force ratios.

The subset of FB data set is depicted in Figure 7. In this figure there appears to be an increase in the final force ratio as the frontage size increases. There also tend to be many more engagements taking place at smaller frontage sizes than were present in the other two data sets. This occurred because command and control rules in FORCEM allowed a unit to engage all enemy units in range. These rules were changed because it was believed so many small engagements are not

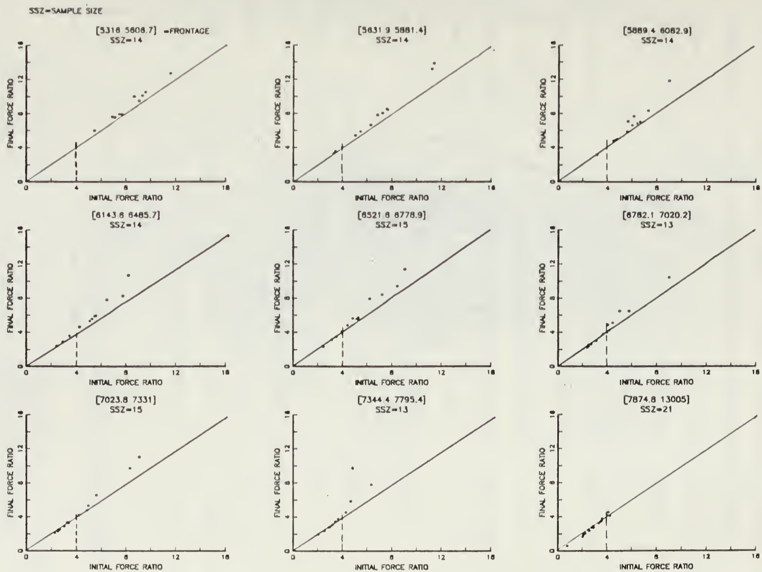


Figure 6. Conditional Scatter-plot of Final vs. Initial Force Ratios Conditioned on Frontage (Data=FR)

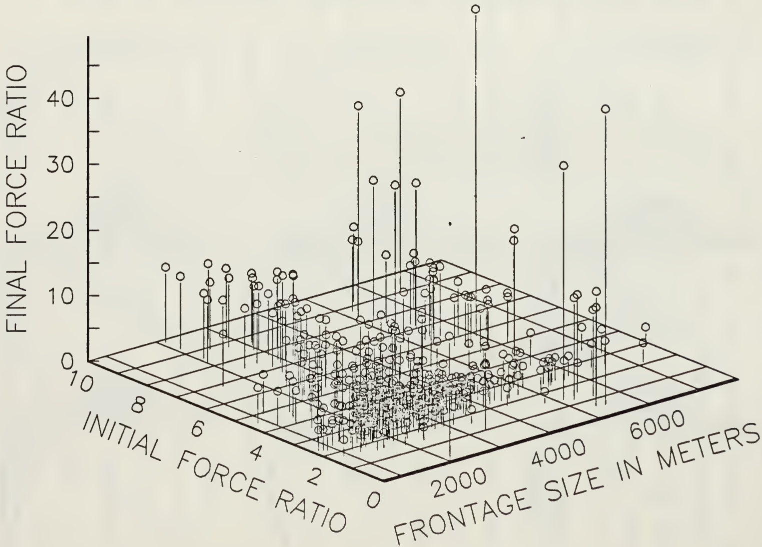


Figure 7. Final Force Ratio Plotted as a Function of Initial Force Ratio and Frontage Size (N=408), Data=FB

desirable in a theater level model. The graphs in Figure 8 display the empirical densities of both the initial force ratio and final force ratio against different frontage sizes. There is a noticeable difference between the density of the final force ratio and the initial force ratio with respect to the thickness of their tails at the higher ratios. A lower overall range of initial force ratios leads to a noticeable migration to lower final force ratios. The range of initial and final force ratios is lower than the range for the FR and FC data. The mode of distributions occurs at lower initial force ratios and is higher compared to the other data sets. Also, there is more spreading of the data in the final force ratio. Figure 9 indicates the same graph as seen in Figure 3 and 6 except with the subset of FB data set. As with the other graphs, the blue force does not consistently lose a higher combat worth proportion relative to the red force until the initial force ratio is higher than a 4 to 1. In fact, where the frontage size is between 2400 and 7650 meters the red force needs at least a 5 to 1 ratio to win consistently. Although in this figure it can be seen that narrow frontage does not point to high initial and final force ratio, it also points to many small engagements which is not considered desirable in a theater level model.

More specifically, in the investigation of changes of frontage size and how they affect the final force ratio in the FORCEM, the following information has been revealed:

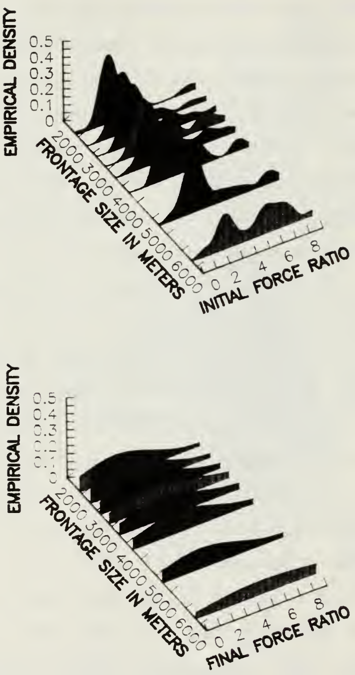


Figure 8. Empirical Densities of Initial and Final Force Ratios Conditioned on Frontage Size (N=408) (Data=FB)

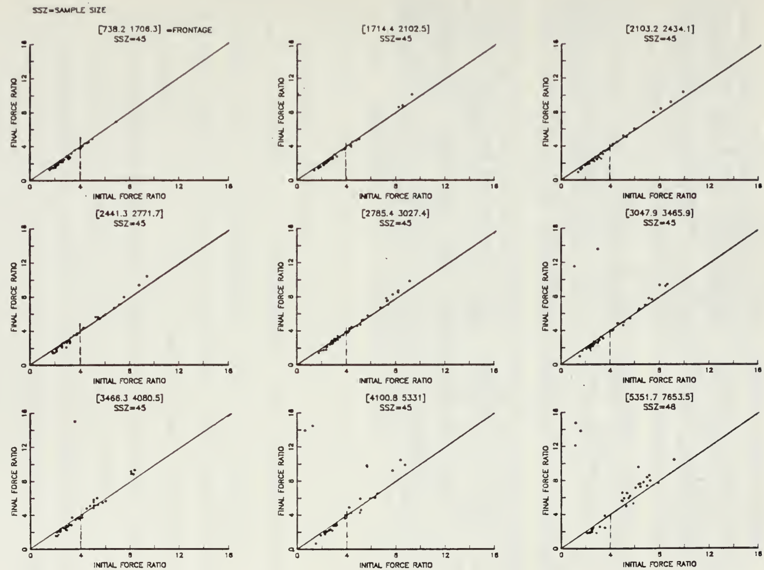


Figure 9. Conditioned Scatter-plots of Final vs. Initial Force Ratios Conditioned on Frontage (Data=FB)

- Changing of frontage sizes show no significant affect in the final force ratio. There is a slight tendency for the final force ratio to increase at all frontage sizes, but the increase is not significant at the larger frontage sizes as would be expected. Thus, frontage changes have no significant affect on final force ratio.

- In order for the red force to consistently lose a lower combat worth proportion relative to the blue force over all the frontage sizes, the red force must have an initial force ratio higher than 4 to 1. Decisive outcomes do not take place when the attacker has a 10:1 ratio and this is definitely opposite to what is expected in a model.

- The combat mechanism used in generating data set FB produces a large number of small frontage, low force ratio (and, therefore, small combat worth) engagements, which are far from the division vs. brigade type engagements which are desired in a theater level model. They dissipate the attackers force unrealistically.

- Between the data set FB and the data sets FC and FR there is a different response: In FB there are many low initial and final force ratios data points at all ranges, where as in FC and FR the initial and final force ratios data points change distribution with range. Also there are larger engagements in the FC and FR data sets. These last two observations are positive results for the model in that they are consistent with what is expected.

IV. ABSENCE OF A MAJOR WEAPON SYSTEM

A major weapon system is any weapon system which, when removed from a unit, will degrade the ability of that unit to accomplish its mission. The major weapon system which will be investigated in this thesis is the main battle tank. The main battle tank is the backbone of the heavy combat divisions deployed throughout the world. The question which will be investigated is: How does the absence of an important weapon system affect the performance of ATCAL in FORCEM results?

A subset of the FR data set which comprises all engagements where the forces were in the most intense conflict during the day was used to analyze the question.

The analysis was undertaken using two different approaches. In the first, a single set of data was analyzed to investigate the effect of removing an entire system type. However, problems were encountered with this portion of the analysis, so it was abandoned in favor of an approach which used two subsets of data with different force mixes to address the question. For the sake of completeness, the first method is described briefly before describing the second method.

In the first method, the final force ratios (red/blue) were calculated for a number of different engagements from the FORCEM results with all the weapons systems present. Then the

main battle tank was removed from the blue units and the final force ratios were recalculated.

The final force ratio without the main battle tank minus the final force ratio with the main battle tank was calculated for all the engagements. This difference was then plotted against the fraction of the force made up by the main battle tank. The logarithms of the force ratio differences were plotted to reduce the crowding of the points [Ref. 6:p. 178].

A linear regression of the log of the differences vs. the fraction of the force made up by the main battle tank was performed and the fitted line was drawn on the graph, using the least square and the scatter plot functions of GRAFSTAT. This was done to check if there existed an association between the log of the differences of the final force ratios and the fraction of the force made up by the main battle tank. The graph in Figure 10 displays the scatter plot with the fitted line. There does appear to be an increase in the difference of the final force ratios as the fraction of the force made up by the main battle tank increases. This increase cannot be proven statistically significant for at least two reasons: the plot of the residuals was not normal and the values for the correlation coefficient were low. This indicates a lack of confidence in the association seen; and is probably due to the relatively large variance in the predicted variable and possibly due to the lack of a linear relationship.

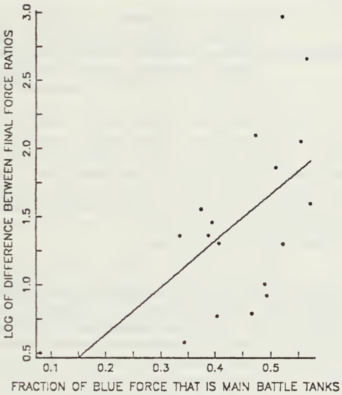


Figure 10. Scatter-plot of the Log of Difference Between Final Force Ratios vs. the Fractions of the Blue Force that is made up of Main Battle Tanks with a Least Squares Fit for the Straight Line (N=18) (Data=Subset of FR)

In order to prove statistically that the absence of an important weapon system affects the performance of ATCAL in FORCEM results a second analytical approach using the Chi-Square test for differences was undertaken [Ref. 7:p. 153]. To perform this test the tank loss fraction of the blue units with a certain main battle tank was calculated and it was also calculated for the blue units without that main battle tank. The tank loss fraction was calculated in the following manner:

$$\text{Tank loss fraction} = \frac{\text{Total combat worth of blue tanks lost}}{\text{Total combat worth of the blue tanks in the engagement}}$$

for both sets of blue units. The tank loss fraction was also calculated for red units with a certain red main battle tank and red units without that red main battle tank. Contingency tables were developed as follows:

TANK LOSS FRACTION OF RED UNITS	BLUE UNITS WITH MAIN BATTLE TANK	BLUE UNITS WITHOUT MAIN BATTLE TANK
0 - .1	f*	f
.1 - ∞	f	f

TANK LOSS FRACTION OF BLUE UNITS	BLUE UNITS WITH MAIN BATTLE TANK	BLUE UNITS WITHOUT MAIN BATTLE TANK
0 - .25	f	f
.25 - ∞	f	f

* Actual frequency will be placed in these cells

The first contingency table was used to test if there was a difference between the lethality of blue units with the main battle tank and the blue units without the main battle tanks. The second contingency table was used to test if there was a difference between the vulnerably of the blue units with the main battle tank and the blue units without the main battle tank. Similar contingency tables were used to test the red units to see if any difference existed between the units with the main battle tank and those without the main battle tank. The complete calculations are located in Appendix B for both the blue and red units. The results were that there was no significant difference found between the lethality of the red units with the main battle tank and those without the main battle tank. There was a significant difference between the lethality of the blue units with the main battle tank and the blue units without the main battle tank. There was no significant difference between the vulnerability of the blue units with the main battle tank and the blue units without the main battle tank. No difference was found between the vulnerability of the red units either.

These findings by themselves do not answer the question of whether the absence of a major weapon system affects the performance of ATCAL in FORCEM results, because certain characteristics of the forces or systems themselves may confound these results. For example, the tank absent from the blue force may actually be a more potent killer but have the

same vulnerability as the other tanks in the blue force, or it may be present in a force with some other potent killer which makes it appear more lethal in the red tank loss column. On the red side, the designated tank may be present or absent in numbers too small to affect the overall kill or vunlerability rates.

In order to qualitatively assess whether differences in force composition may be influencing the analysis, the star symbol plots will be used. The star symbol is an interesting way of graphing multivariate data so that you can visually compare the different units. Each star represents one blue unit. There will be two sets, the blue units with the designated main battle tank present and those without the main battle tank. For this study the rays will represent the number of tanks, helicopters (helos), and anti-armor guided missiles (ATGMS) the unit has. The helicopters and anti-armor guided missiles are the two other major tank killing systems in the battlefield within the model. [Ref. 6:pp. 155-159] Figure 11 shows a representative of the star symbol plot and it has each of the rays labeled with the weapon system it represents. This star symbol plot is the legend for the star symbol plots found in Figures 12 and 13.

The force composition may be compared both within and across the data sets using Figures 12 and 13. For instance, the units represented in Figure 12 appear to be reasonably homogeneous with a stable fraction of tanks, but with some

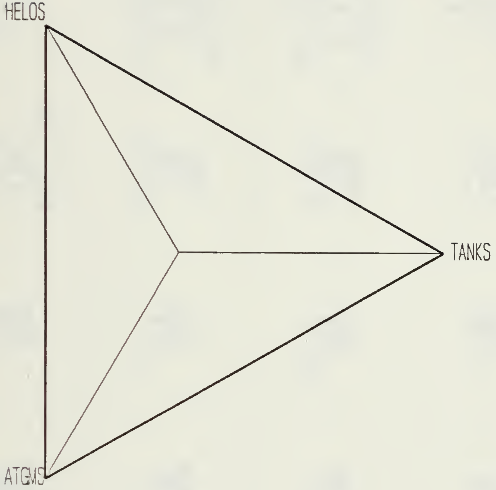


Figure 11. Star Symbol Plot with the Rays Labeled to Represent the Star Symbol Plots in Figures 12 and 13.

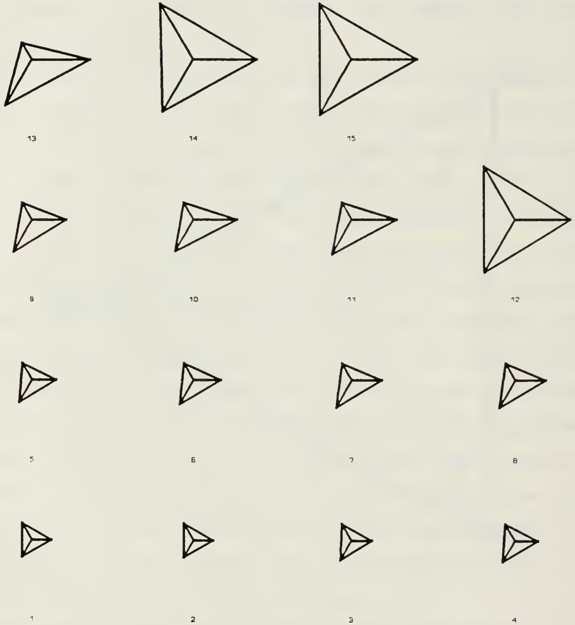


Figure 12. Star Symbol Plot of the Blue Units with the Main Battle Tank

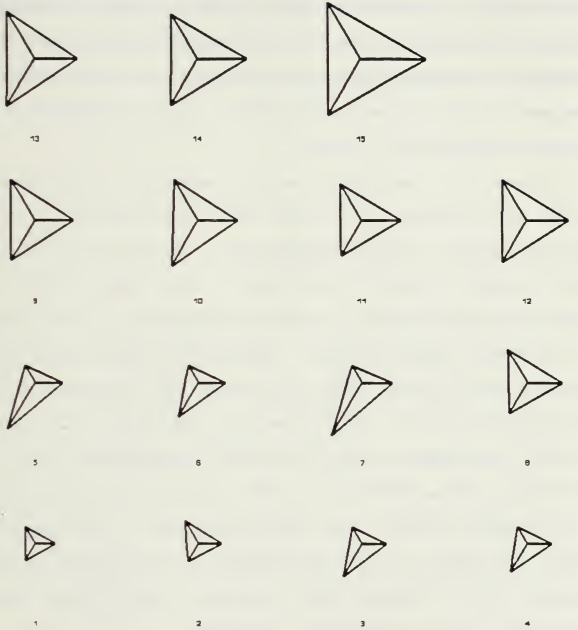


Figure 13. Star Symbol Plot of the Blue Units Without the Main Battle Tank

fluctuation in the fraction of helicopters and ATGMS present. In Figure 13 there appears to be a stable tank fraction present, but more variability in the helicopter and ATGM fractions. Between Figures 12 and 13 these combined differences, while not excessive, may be enough to influence the loss fractions of tanks for the engagements used in this analysis. Because the data provided does not specify precisely which systems killed which, it is not possible to take the analysis any further.

No hard line conclusions can be drawn about the model due to the small sample size of data that was available to test for differences in ATCAL performance. However, the results seem to point to the fact that when a major weapon system is absent some other system or systems compensate for the absence of the major weapon system. This observation needs more definite data to study then is available for this study. From the results of the Chi-Square test for difference only one of the four tests made was able to reject the hypothesis that no difference existed between the units with the main battle tank and the units without the main battle tank. The graph in Figure 10 tends to show an increase in difference as the fraction of the force made up by the main battle tank increases, but no statistical significance can be proven.

The results from the Chi-Square tests for difference show less effect in the data due to the absence of a major weapon system than might be expected, but in light of other factors,

it is hard to attribute this observation to the performance of ATCAL. The small sample size available, differences in the forces involved, and lack of specific killer victim information all influence this analysis and weaken any conclusion with respect to ATCAL performance. However, the techniques employed here would be appropriate in conducting a controlled analysis of this question when more detailed data is available.

V. CONCLUSIONS

A. SUMMARY

The purpose of this thesis was to analyze an attrition process in the context of its theater model. A graphical data analysis of the sensitivities of ATCAL with respect to FORCEM operational data was performed. Given the ATCAL results from various FORCEM runs, the sensitivity of ATCAL within the FORCEM model to the effects of frontage of engagement and presence of important weapon systems was investigated. The following observations have been made:

- Changing of frontage size show no significant effect on the final force ratio.

- In order for the red force to consistently lose a lower combat worth proportion relative to the blue force over all the frontage sizes, the red force must have an initial force ratio higher than 4:1. Decisive outcomes do not take place when the attacker has a 10:1 ratio.

- Large numbers of small frontage, low force ratio engagements, which are far from the division versus brigade type engagements which are wanted in a theater level model, dissipate the attacker force.

- Restricting the number of engagements each unit can be involved in per time period so that the number of extremely small engagements are reduced causes the initial and final force ratios data points to change distribution with range. Also there are larger engagements taking place which are positive results for the model.

- Absence of a major weapon system seemed to have only limited effect on the ATCAL results tested, but several factors may have influenced these results. More detail and control is required in the data to be able to draw ATCAL specific conclusions.

B. POSSIBLE CORRECTIONS

The method which was used in data sets FR and FC so that the unit does not get involved in too many engagements in a time period should be retained in FORCEM so that the number of small engagements will be reduced.

The process for calculating the engagement frontage in FORCEM requires further testing and possible modification so that frontage will have a discernible effect on the outcome of battle.

C. ADDITIONAL POSSIBLE AREAS OF STUDY

If there is additional interest in the sensitivities of ATCAL with respect to FORCEM, topics remaining open for research are:

- The effect of removing a major weapon system needs to be further studied with more detailed data.
- What effect does the wide variation in size and force mix of engagements have on the performance of ATCAL?
- What important change in response is due to special systems such as close air support?

In order to perform any of the above mentioned areas of research basic knowledge of both the ATCAL and FORCEM model is recommended. Also close coordination with CAA is necessary to provide data with suitable detail to conduct the desired analysis.

APPENDIX A

DATA

```
1          FORCEM BASIC DATA (FB)

4          PROC UNIVARIATE DATA = DD1.E6;
5             VAR TIME--FINARA;
                                        UNIVARIATE

VARIABLE=TIME

                                         MOMENTS

                  N                 47487   SUM WGTS          47487
                  MEAN            4.87355   SUM              231431
                  STD DEV          2.9125   VARIANCE        8.48267
                  SKEWNESS     -0.0711565   KURTOSIS         -1.155
                  USS             1530697   CSS              402808
                  CV              59.7613   STD MEAN      0.0133653
                  T:MEAN=0        364.642   PROB>|T|         0.0001
                  SGN RANK      486720813   PROB>|S|         0.0001
                  NUM ¬= 0          44123

             QUANTILES(DEF=4)                                               EXTREMES

100% MAX        9.5       99%          9.5                             LOWEST    HIGHEST
 75% Q3         7.5       95%            9                                  0        9.5
 50% MED          5       90%            9                                  0        9.5
 25% Q1         2.5       10%          0.5                                  0        9.5
  0% MIN          0        5%            0                                  0        9.5
                           1%            0                                  0        9.5
 RANGE          9.5
 Q3-Q1            5
 MODE             0
                                        UNIVARIATE

VARIABLE=BRD

                                         MOMENTS

                  N                 47487   SUM WGTS          47487
                  MEAN            12.6844   SUM              602342
                  STD DEV          5.4131   VARIANCE        29.3017
                  SKEWNESS       0.105552   KURTOSIS       -1.55705
                  USS             9031740   CSS             1391420
                  CV              42.6754   STD MEAN      0.0248404
                  T:MEAN=0        510.634   PROB>|T|         0.0001
                  SGN RANK      563765664   PROB>|S|         0.0001
                  NUM ¬= 0          47487

             QUANTILES(DEF=4)                                               EXTREMES

100% MAX         20       99%           20                             LOWEST    HIGHEST
 75% Q3          18       95%           20                                  6         20
 50% MED         10       90%           20                                  6         20
 25% Q1           6       10%            6                                  6         20
  0% MIN          6        5%            6                                  6         20
                           1%            6                                  6         20
 RANGE           14
 Q3-Q1           12
 MODE             6
```

UNIVARIATE

VARIABLE=FRONT

MOMENTS

N	47487	SUM WGTS	47487
MEAN	326.653	SUM	15511794
STD DEV	148.537	VARIANCE	22063.2
SKEWNESS	1.01984	KURTOSIS	1.38137
USS	6114676497	CSS	1047694739
CV	45.4723	STD MEAN	0.681628
T:MEAN=0	479.226	PROB>|T|	0.0001
SGN RANK	563765664	PROB>|S|	0.0001
NUM ¬= 0	47487		

QUANTILES(DEF=4)

100% MAX	1046.66	99%	756.51
75% Q3	404.08	95%	603.31
50% MED	301.59	90%	543.1
25% Q1	214.29	10%	163.66
0% MIN	48.16	5%	134.83
		1%	95.14
RANGE	998.5		
Q3-Q1	189.79		
MODE	190.48		

EXTREMES

LOWEST	HIGHEST
48.16	1046.66
48.16	1046.66
48.16	1046.66
48.16	1046.66
48.16	1046.66

UNIVARIATE

VARIABLE=AST

MOMENTS

N	47487	SUM WGTS	47487
MEAN	61.5547	SUM	2923050
STD DEV	28.0058	VARIANCE	784.324
SKEWNESS	-0.39991	KURTOSIS	-0.917641
USS	217171980	CSS	37244393
CV	45.4974	STD MEAN	0.128517
T:MEAN=0	478.962	PROB>|T|	0.0001
SGN RANK	563765664	PROB>|S|	0.0001
NUM ¬= 0	47487		

QUANTILES(DEF=4)

100% MAX	102	99%	102
75% Q3	87	95%	98
50% MED	65	90%	96
25% Q1	40	10%	21
0% MIN	4	5%	5
		1%	4
RANGE	98		
Q3-Q1	47		
MODE	5		

EXTREMES

LOWEST	HIGHEST
4	102
4	102
4	102
4	102
4	102

UNIVARIATE

VARIABLE=OHAND

MOMENTS

N	47487	SUM WGTS	47487
MEAN	34.6687	SUM	1646311
STD DEV	153.033	VARIANCE	23419.1
SKEWNESS	13.4161	KURTOSIS	257.69
USS	1169154656	CSS	1112079265
CV	441.416	STD MEAN	0.70226
T:MEAN=0	49.3673	PROB>|T|	0.0001
SGN RANK	563765664	PROB>|S|	0.0001
NUM ¬= 0	47487		

QUANTILES(DEF=4)

100% MAX	5186	99%	569.733
75% Q3	17.67	95%	108.736
50% MED	7.02	90%	48
25% Q1	2.78	10%	1.02
0% MIN	0.03	5%	0.57
		1%	0.21
RANGE	5185.97		
Q3-Q1	14.89		
MODE	4.32		

EXTREMES

LOWEST	HIGHEST
0.03	4351
0.03	4429
0.03	4784
0.03	5148
0.03	5186

UNIVARIATE

VARIABLE=HITS

MOMENTS

N	47487	SUM WGTS	47487
MEAN	1.83319	SUM	87052.7
STD DEV	5.74933	VARIANCE	33.0548
SKEWNESS	14.948	KURTOSIS	586.03
USS	1729225	CSS	1569641
CV	313.625	STD MEAN	0.0263834
T:MEAN=0	69.4828	PROB>|T|	0.0001
SGN RANK	404221078	PROB>|S|	0.0001
NUM ¬= 0	40210		

QUANTILES(DEF=4)

100% MAX	395.06	99%	23.8736
75% Q3	1.29	95%	8.56
50% MED	0.27	90%	4.57
25% Q1	0.03	10%	0
0% MIN	0	5%	0
		1%	0
RANGE	395.06		
Q3-Q1	1.26		
MODE	0		

EXTREMES

LOWEST	HIGHEST
0	138.55
0	145.23
0	158.21
0	184.94
0	395.06

```
                                        UNIVARIATE
VARIABLE=TOASCW

                                          MOMENTS

                     N                 47487   SUM WGTS         47487
                     MEAN            2.29222   SUM             108851
                     STD DEV         5.47611   VARIANCE       29.9878
                     SKEWNESS        5.76316   KURTOSIS       49.4148
                     USS             1673512   CSS            1424002
                     CV                238.9   STD MEAN     0.0251296
                     T:MEAN=0        91.2161   PROB>|T|        0.0001
                     SGN RANK      423402641   PROB>|S|        0.0001
                     NUM ^= 0          41153

              QUANTILES(DEF=4)                                              EXTREMES

100% MAX        99.41       99%          26.98                           LOWEST    HIGHEST
 75% Q3          1.85       95%          10.97                                0      90.73
 50% MED         0.51       90%           5.93                                0      93.84
 25% Q1          0.07       10%              0                                0       94.9
  0% MIN            0        5%              0                                0         97
                              1%              0                                0      99.41
 RANGE          99.41
 Q3-Q1           1.78
 MODE               0
```

```
                                        UNIVARIATE
VARIABLE=LOASCW

                                          MOMENTS

                     N                 47487   SUM WGTS         47487
                     MEAN           0.305974   SUM            14529.8
                     STD DEV          1.3988   VARIANCE       1.95664
                     SKEWNESS        15.6969   KURTOSIS       423.446
                     USS             97358.7   CSS              92913
                     CV              457.162   STD MEAN    0.00641901
                     T:MEAN=0        47.6669   PROB>|T|        0.0001
                     SGN RANK      186001863   PROB>|S|        0.0001
                     NUM ^= 0          27276

              QUANTILES(DEF=4)                                              EXTREMES

100% MAX        74.68       99%         5.0336                           LOWEST    HIGHEST
 75% Q3          0.12       95%           1.37                                0      39.19
 50% MED         0.01       90%           0.59                                0      39.47
 25% Q1             0       10%              0                                0      46.32
  0% MIN            0        5%              0                                0      52.89
                              1%              0                                0      74.68
 RANGE          74.68
 Q3-Q1           0.12
 MODE               0
```

UNIVARIATE

VARIABLE=TOBLCW

MOMENTS

N	47372	SUM WGTS	47372
MEAN	24.0796	SUM	1140701
STD DEV	26.6573	VARIANCE	710.612
SKEWNESS	2.93868	KURTOSIS	12.5386
USS	61130103	CSS	33662419
CV	110.705	STD MEAN	0.122477
T:MEAN=0	196.605	PROB>|T|	0.0001
SGN RANK	561038439	PROB>|S|	0.0001
NUM ¬= 0	47372		

QUANTILES(DEF=4)

100% MAX	275.11	99%	135.172
75% Q3	28.38	95%	77.92
50% MED	14.25	90%	54.187
25% Q1	8.48	10%	5.65
0% MIN	1.42	5%	4.27
		1%	2.83
RANGE	273.69		
Q3-Q1	19.9		
MODE	8.11		

EXTREMES

LOWEST	HIGHEST
1.42	275.11
1.42	275.11
1.42	275.11
1.42	275.11
1.42	275.11

MISSING VALUE	.
COUNT	115
% COUNT/NOBS	0.24

UNIVARIATE

VARIABLE=LOBLCW

MOMENTS

N	47372	SUM WGTS	47372
MEAN	2.07779	SUM	98429.2
STD DEV	3.02657	VARIANCE	9.16014
SKEWNESS	4.40846	KURTOSIS	29.8451
USS	638441	CSS	433925
CV	145.663	STD MEAN	0.0139056
T:MEAN=0	149.421	PROB>|T|	0.0001
SGN RANK	561038439	PROB>|S|	0.0001
NUM ¬= 0	47372		

QUANTILES(DEF=4)

100% MAX	35.75	99%	15.44
75% Q3	2.4	95%	6.98
50% MED	1.09	90%	4.93
25% Q1	0.49	10%	0.21
0% MIN	0.01	5%	0.13
		1%	0.05
RANGE	35.74		
Q3-Q1	1.91		
MODE	0.18		

EXTREMES

LOWEST	HIGHEST
0.01	35.75
0.01	35.75
0.01	35.75
0.01	35.75
0.01	35.75

MISSING VALUE	.
COUNT	115
% COUNT/NOBS	0.24

UNIVARIATE

VARIABLE=INITRA

MOMENTS

N	47372	SUM WGTS	47372
MEAN	2.74719	SUM	130140
STD DEV	1.82658	VARIANCE	3.33639
SKEWNESS	1.43869	KURTOSIS	2.00127
USS	515567	CSS	158048
CV	66.489	STD MEAN	0.00839223
T:MEAN=0	327.349	PROB>|T|	0.0001
SGN RANK	561038439	PROB>|S|	0.0001
NUM ¬= 0	47372		

QUANTILES(DEF=4)

100% MAX	9.86	99%	8.85
75% Q3	3.35	95%	6.85
50% MED	2.27	90%	5.39
25% Q1	1.52	10%	0.93
0% MIN	0.05	5%	0.66
		1%	0.31
RANGE	9.81		
Q3-Q1	1.83		
MODE	1.53		

EXTREMES

LOWEST	HIGHEST
0.05	9.86
0.05	9.86
0.05	9.86
0.05	9.86
0.05	9.86

MISSING VALUE .
COUNT 115
% COUNT/NOBS 0.24

UNIVARIATE

VARIABLE=FINARA

MOMENTS

N	47372	SUM WGTS	47372
MEAN	2.97446	SUM	140906
STD DEV	3.21138	VARIANCE	10.313
SKEWNESS	5.86166	KURTOSIS	57.2359
USS	907657	CSS	488536
CV	107.965	STD MEAN	0.0147547
T:MEAN=0	201.594	PROB>|T|	0.0001
SGN RANK	561038439	PROB>|S|	0.0001
NUM ¬= 0	47372		

QUANTILES(DEF=4)

100% MAX	48.71	99%	14.46
75% Q3	3.33	95%	7.97
50% MED	2.13	90%	5.83
25% Q1	1.45	10%	0.88
0% MIN	0.08	5%	0.63
		1%	0.26
RANGE	48.63		
Q3-Q1	1.88		
MODE	1.97		

EXTREMES

LOWEST	HIGHEST
0.08	48.71
0.08	48.71
0.08	48.71
0.08	48.71
0.08	48.71

MISSING VALUE .
COUNT 115
% COUNT/NOBS 0.24

```
          FORCEM RESTRICTED ENGAGEMENTS DATA (FR)


              INPUT  TIME     3-5
                     BRD      6-8
                     BLUE    10-15
                     FBLUE   17-21
                     RED     22-28
                     FRED    30-34
                     FRONT   35-42
                     AST     44-46
                     OHAND   48-54
                     HITS    56-62
                     GLOBI   66-73
                     TOASCW  74-80
                     LOASCW  82-87
                     TOBLCW  89-94
                     LOBLCW  96-101
                     INITRA  103-107
                     FINARA  109-114;
            PROC UNIVARIATE DATA= E52 ;
               VAR TIME--FINARA;
```

UNIVARIATE

```
VARIABLE=TIME
```

MOMENTS

N	19985	SUM WGTS	19985
MEAN	4.47245	SUM	89382
STD DEV	2.60391	VARIANCE	6.78035
SKEWNESS	0.0827012	KURTOSIS	-0.960139
USS	535255	CSS	135499
CV	58.2211	STD MEAN	0.0184193
T:MEAN=0	242.813	PROB>|T|	0.0001
SGN RANK	91934538	PROB>|S|	0.0001
NUM ¬= 0	19176		

QUANTILES(DEF=4)

100% MAX	9.5	99%	9.5
75% Q3	6.5	95%	9
50% MED	4.5	90%	8
25% Q1	2.5	10%	1
0% MIN	0	5%	0.5
		1%	0
RANGE	9.5		
Q3-Q1	4		
MODE	5		

EXTREMES

LOWEST	HIGHEST
0	9.5
0	9.5
0	9.5
0	9.5
0	9.5

UNIVARIATE

VARIABLE=FBLUE

MOMENTS

N	19985	SUM WGTS	19985
MEAN	0.405909	SUM	8112.08
STD DEV	0.294719	VARIANCE	0.0868594
SKEWNESS	0.906959	KURTOSIS	-0.284561
USS	5028.56	CSS	1735.8
CV	72.6073	STD MEAN	0.00208476
T:MEAN=0	194.703	PROB>|T|	0.0001
SGN RANK	99855053	PROB>|S|	0.0001
NUM ¬= 0	19985		

QUANTILES(DEF=4)

100% MAX	1	99%	1
75% Q3	0.54	95%	1
50% MED	0.333	90%	1
25% Q1	0.159	10%	0.095
0% MIN	0.016	5%	0.079
		1%	0.048
RANGE	0.984		
Q3-Q1	0.381		
MODE	1		

EXTREMES

LOWEST	HIGHEST
0.016	1
0.016	1
0.016	1
0.016	1
0.016	1

UNIVARIATE

VARIABLE=RED

MOMENTS

N	19985	SUM WGTS	19985
MEAN	103341	SUM	2065273784
STD DEV	2144.63	VARIANCE	4599433
SKEWNESS	0.343905	KURTOSIS	-1.38798
USS	2.135E+14	CSS	9.192E+10
CV	2.07529	STD MEAN	15.1705
T:MEAN=0	6811.98	PROB>|T|	0.0001
SGN RANK	99855053	PROB>|S|	0.0001
NUM ¬= 0	19985		

QUANTILES(DEF=4)

100% MAX	107025	99%	107025
75% Q3	105015	95%	107024
50% MED	102025	90%	107021
25% Q1	101034	10%	101032
0% MIN	101031	5%	101031
		1%	101031
RANGE	5994		
Q3-Q1	3981		
MODE	101033		

EXTREMES

LOWEST	HIGHEST
101031	107025
101031	107025
101031	107025
101031	107025
101031	107025

UNIVARIATE

VARIABLE=FRED

MOMENTS

N	19985	SUM WGTS	19985
MEAN	0.871713	SUM	17421.2
STD DEV	0.229705	VARIANCE	0.0527645
SKEWNESS	-1.39698	KURTOSIS	0.344504
USS	16240.7	CSS	1054.45
CV	26.351	STD MEAN	0.00162487
T:MEAN=0	536.481	PROB>|T|	0.0001
SGN RANK	99855053	PROB>|S|	0.0001
NUM ¬= 0	19985		

QUANTILES(DEF=4)

100% MAX	1	99%	1
75% Q3	1	95%	1
50% MED	1	90%	1
25% Q1	0.762	10%	0.476
0% MIN	0.095	5%	0.381
		1%	0.254
RANGE	0.905		
Q3-Q1	0.238		
MODE	1		

EXTREMES

LOWEST	HIGHEST
0.095	1
0.095	1
0.095	1
0.095	1
0.095	1

UNIVARIATE

VARIABLE=FRONT

MOMENTS

N	19985	SUM WGTS	19985
MEAN	6860.76	SUM	137112300
STD DEV	1730.42	VARIANCE	2994363
SKEWNESS	0.879289	KURTOSIS	1.15048
USS	1.001E+12	CSS	5.984E+10
CV	25.222	STD MEAN	12.2405
T:MEAN=0	560.495	PROB>|T|	0.0001
SGN RANK	99855053	PROB>|S|	0.0001
NUM ¬= 0	19985		

QUANTILES(DEF=4)

100% MAX	13005	99%	11953.4
75% Q3	7738.3	95%	10119.1
50% MED	6428.6	90%	9170
25% Q1	5716.3	10%	5254.1
0% MIN	2448.1	5%	4591
		1%	3172.2
RANGE	10556.9		
Q3-Q1	2022		
MODE	11635		

EXTREMES

LOWEST	HIGHEST
2448.1	13005
2448.1	13005
2448.1	13005
2448.1	13005
2448.1	13005

UNIVARIATE

VARIABLE=HITS

MOMENTS

N	19985	SUM WGTS	19985
MEAN	3.4619	SUM	69186.1
STD DEV	7.33618	VARIANCE	53.8195
SKEWNESS	4.17319	KURTOSIS	23.8599
USS	1315045	CSS	1075530
CV	211.912	STD MEAN	0.0518941
T:MEAN=0	66.7109	PROB>|T|	0.0001
SGN RANK	78921014	PROB>|S|	0.0001
NUM ¬= 0	17767		

QUANTILES(DEF=4)

100% MAX	112.25	99%	36.7542
75% Q3	3.09	95%	17.76
50% MED	0.69	90%	9.784
25% Q1	0.07	10%	0
0% MIN	0	5%	0
		1%	0
RANGE	112.25		
Q3-Q1	3.02		
MODE	0		

EXTREMES

LOWEST	HIGHEST
0	80.17
0	81.26
0	85.39
0	85.68
0	112.25

UNIVARIATE

VARIABLE=GLOBI

MOMENTS

N	19985	SUM WGTS	19985
MEAN	0.196315	SUM	3923.36
STD DEV	0.277462	VARIANCE	0.076985
SKEWNESS	2.52159	KURTOSIS	7.50073
USS	2308.68	CSS	1538.47
CV	141.335	STD MEAN	0.00196269
T:MEAN=0	100.024	PROB>|T|	0.0001
SGN RANK	79856564	PROB>|S|	0.0001
NUM ¬= 0	17872		

QUANTILES(DEF=4)

100% MAX	1.475	99%	1.475
75% Q3	0.2796	95%	0.7805
50% MED	0.0939	90%	0.5272
25% Q1	0.0092	10%	0
0% MIN	0	5%	0
		1%	0
RANGE	1.475		
Q3-Q1	0.2704		
MODE	0		

EXTREMES

LOWEST	HIGHEST
0	1.475
0	1.475
0	1.475
0	1.475
0	1.475

UNIVARIATE

VARIABLE=TOASCW

MOMENTS

N	19985	SUM WGTS	19985
MEAN	7.19784	SUM	143849
STD DEV	15.7364	VARIANCE	247.634
SKEWNESS	4.13521	KURTOSIS	22.1199
USS	5984110	CSS	4948709
CV	218.626	STD MEAN	0.111315
T:MEAN=0	64.662	PROB>|T|	0.0001
SGN RANK	79802956	PROB>|S|	0.0001
NUM ¬= 0	17866		

QUANTILES(DEF=4)

100% MAX	188	99%	82.8442
75% Q3	5.66	95%	36.951
50% MED	1.46	90%	22.13
25% Q1	0.24	10%	0
0% MIN	0	5%	0
		1%	0
RANGE	188		
Q3-Q1	5.42		
MODE	0		

EXTREMES

LOWEST	HIGHEST
0	157.63
0	164
0	164
0	186
0	188

UNIVARIATE

VARIABLE=LOASCW

MOMENTS

N	19985	SUM WGTS	19985
MEAN	0.60957	SUM	12182.3
STD DEV	1.96689	VARIANCE	3.86866
SKEWNESS	6.75512	KURTOSIS	64.8071
USS	84737.3	CSS	77311.4
CV	322.669	STD MEAN	0.0139132
T:MEAN=0	43.8122	PROB>|T|	0.0001
SGN RANK	42481065	PROB>|S|	0.0001
NUM ¬= 0	13035		

QUANTILES(DEF=4)

100% MAX	40.91	99%	10.37
75% Q3	0.28	95%	3.167
50% MED	0.03	90%	1.46
25% Q1	0	10%	0
0% MIN	0	5%	0
		1%	0
RANGE	40.91		
Q3-Q1	0.28		
MODE	0		

EXTREMES

LOWEST	HIGHEST
0	28.74
0	30.25
0	34.91
0	35.9
0	40.91

UNIVARIATE

VARIABLE=TOBLCW

MOMENTS

N	19985	SUM WGTS	19985
MEAN	74.0713	SUM	1480314
STD DEV	84.1156	VARIANCE	7075.43
SKEWNESS	2.97926	KURTOSIS	10.8913
USS	251044164	CSS	141395447
CV	113.56	STD MEAN	0.59501
T:MEAN=0	124.487	PROB>|T|	0.0001
SGN RANK	99855053	PROB>|S|	0.0001
NUM ¬= 0	19985		

QUANTILES(DEF=4)

100% MAX	565.2	99%	456.04
75% Q3	89.21	95%	235.55
50% MED	47.26	90%	149.9
25% Q1	24.96	10%	14.44
0% MIN	3.07	5%	11.14
		1%	6.33
RANGE	562.13		
Q3-Q1	64.25		
MODE	16.22		

EXTREMES

LOWEST	HIGHEST
3.07	565.2
3.07	565.2
3.07	565.2
3.07	565.2
3.07	565.2

UNIVARIATE

VARIABLE=LOBLCW

MOMENTS

N	19985	SUM WGTS	19985
MEAN	5.57662	SUM	111449
STD DEV	5.81112	VARIANCE	33.7691
SKEWNESS	1.99941	KURTOSIS	4.47509
USS	1296349	CSS	674842
CV	104.205	STD MEAN	0.0411062
T:MEAN=0	135.664	PROB>|T|	0.0001
SGN RANK	99855053	PROB>|S|	0.0001
NUM ¬= 0	19985		

QUANTILES(DEF=4)

100% MAX	34.62	99%	26.08
75% Q3	7.34	95%	18.56
50% MED	3.38	90%	13.48
25% Q1	1.68	10%	0.98
0% MIN	0.06	5%	0.63
		1%	0.27
RANGE	34.56		
Q3-Q1	5.66		
MODE	1.68		

EXTREMES

LOWEST	HIGHEST
0.06	34.62
0.06	34.62
0.06	34.62
0.06	34.62
0.06	34.62

UNIVARIATE

VARIABLE=INITRA

MOMENTS

N	19985	SUM WGTS	19985
MEAN	4.37361	SUM	87406.6
STD DEV	4.19756	VARIANCE	17.6195
SKEWNESS	2.41593	KURTOSIS	7.84487
USS	734391	CSS	352108
CV	95.9747	STD MEAN	0.0296924
T:MEAN=0	147.297	PROB>|T|	0.0001
SGN RANK	99855053	PROB>|S|	0.0001
NUM ¬= 0	19985		

QUANTILES(DEF=4)

100% MAX	31.36	99%	22.11
75% Q3	5.58	95%	12.57
50% MED	3.15	90%	9.06
25% Q1	1.57	10%	0.93
0% MIN	0.19	5%	0.63
		1%	0.32
RANGE	31.17		
Q3-Q1	4.01		
MODE	5.46		

EXTREMES

LOWEST	HIGHEST
0.19	31.36
0.19	31.36
0.19	31.36
0.19	31.36
0.19	31.36

UNIVARIATE

VARIABLE=FINARA

MOMENTS

N	19985	SUM WGTS	19985
MEAN	4.86026	SUM	97132.3
STD DEV	5.3951	VARIANCE	29.1071
SKEWNESS	2.89424	KURTOSIS	11.156
USS	1053765	CSS	581676
CV	111.004	STD MEAN	0.0381634
T:MEAN=0	127.354	PROB>|T|	0.0001
SGN RANK	99855053	PROB>|S|	0.0001
NUM ¬= 0	19985		

QUANTILES(DEF=4)

100% MAX	43.4	99%	31.62
75% Q3	6.01	95%	14.53
50% MED	3.17	90%	10.44
25% Q1	1.54	10%	0.92
0% MIN	0.17	5%	0.59
		1%	0.31
RANGE	43.23		
Q3-Q1	4.47		
MODE	1.34		

EXTREMES

LOWEST	HIGHEST
0.17	43.4
0.17	43.4
0.17	43.4
0.17	43.4
0.17	43.4

FORCEM DATA USING CEM's VERSION OF ATCAL (FC)

```
INPUT  TIME     1-5
       BRD      6-8
       BLUE    11-15
       RED     16-21
       FRONT   23-29
       RTNKO   32-37
       RTNKH   39-44
       RTNKK   47-52
       BTNKO   55-59
       BTNKH   62-67
       BTNKK   68-73
       TOBLCW  81-86
       LOBLCW  88-92
       INITRA  95-100
       FINARA 102-106;
PROC UNIVARIATE DATA=CEM;
   VAR TIME--FINARA;
```

UNIVARIATE

VARIABLE=TIME

MOMENTS

N	651	SUM WGTS	651
MEAN	9.29186	SUM	6049
STD DEV	4.95744	VARIANCE	24.5762
SKEWNESS	-0.0296628	KURTOSIS	-1.16771
USS	72181	CSS	15974.5
CV	53.3525	STD MEAN	0.194297
T:MEAN=0	47.8229	PROB>|T|	0.0001
SGN RANK	104491	PROB>|S|	0.0001
NUM ¬= 0	646		

QUANTILES(DEF=4)

100% MAX	17.5	99%	17.5
75% Q3	13.5	95%	17
50% MED	9.5	90%	16
25% Q1	5	10%	2.5
0% MIN	0	5%	1.5
		1%	0.5
RANGE	17.5		
Q3-Q1	8.5		
MODE	5		

EXTREMES

LOWEST	HIGHEST
0	17.5
0	17.5
0	17.5
0	17.5
0	17.5

MISSING VALUE	.
COUNT	1
% COUNT/NOBS	0.15

UNIVARIATE

VARIABLE=RTNKO

MOMENTS

N	651	SUM WGTS	651
MEAN	136.849	SUM	89088.6
STD DEV	67.9056	VARIANCE	4611.17
SKEWNESS	1.28165	KURTOSIS	3.64333
USS	15188933	CSS	2997261
CV	49.6209	STD MEAN	2.66143
T:MEAN=0	51.4193	PROB>|T|	0.0001
SGN RANK	106113	PROB>|S|	0.0001
NUM ¬= 0	651		

QUANTILES(DEF=4)

100% MAX	557	99%	350
75% Q3	176	95%	253.6
50% MED	122	90%	219.8
25% Q1	93.8	10%	65.14
0% MIN	14.6	5%	46.14
		1%	24.66
RANGE	542.4		
Q3-Q1	82.2		
MODE	88		

EXTREMES

LOWEST	HIGHEST
14.6	350
14.6	431
16.2	431
16.3	460
22.5	557

MISSING VALUE .
COUNT 1
% COUNT/NOBS 0.15

UNIVARIATE

VARIABLE=RTNKH

MOMENTS

N	651	SUM WGTS	651
MEAN	11.1261	SUM	7243.1
STD DEV	11.8693	VARIANCE	140.88
SKEWNESS	1.93652	KURTOSIS	4.88096
USS	172159	CSS	91571.7
CV	106.679	STD MEAN	0.465193
T:MEAN=0	23.9172	PROB>|T|	0.0001
SGN RANK	106113	PROB>|S|	0.0001
NUM ¬= 0	651		

QUANTILES(DEF=4)

100% MAX	76.9	99%	57.4
75% Q3	15.7	95%	34.6
50% MED	7.4	90%	28.46
25% Q1	2.4	10%	0.92
0% MIN	0.1	5%	0.5
		1%	0.1
RANGE	76.8		
Q3-Q1	13.3		
MODE	1		

EXTREMES

LOWEST	HIGHEST
0.1	57.4
0.1	57.4
0.1	71.3
0.1	73.6
0.1	76.9

MISSING VALUE .
COUNT 1
% COUNT/NOBS 0.15

UNIVARIATE

VARIABLE=RTNKK

MOMENTS

N	651	SUM WGTS	651
MEAN	5.87358	SUM	3823.7
STD DEV	6.31863	VARIANCE	39.925
SKEWNESS	2.19955	KURTOSIS	7.11849
USS	48410.1	CSS	25951.3
CV	107.577	STD MEAN	0.247646
T:MEAN=0	23.7176	PROB>|T|	0.0001
SGN RANK	105463	PROB>|S|	0.0001
NUM ¬= 0	649		

QUANTILES(DEF=4)

100% MAX	50.1	99%	29.9
75% Q3	7.9	95%	18.1
50% MED	3.7	90%	14.68
25% Q1	1.4	10%	0.6
0% MIN	0	5%	0.3
		1%	0.1
RANGE	50.1		
Q3-Q1	6.5		
MODE	0.6		

EXTREMES

LOWEST	HIGHEST
0	29.9
0	29.9
0.1	39
0.1	39.4
0.1	50.1

MISSING VALUE	.
COUNT	1
% COUNT/NOBS	0.15

UNIVARIATE

VARIABLE=BTNKO

MOMENTS

N	651	SUM WGTS	651
MEAN	58.6608	SUM	38188.2
STD DEV	59.1802	VARIANCE	3502.29
SKEWNESS	1.61457	KURTOSIS	2.33162
USS	4516642	CSS	2276490
CV	100.885	STD MEAN	2.31945
T:MEAN=0	25.2908	PROB>|T|	0.0001
SGN RANK	106113	PROB>|S|	0.0001
NUM ¬= 0	651		

QUANTILES(DEF=4)

100% MAX	278	99%	266.88
75% Q3	85.8	95%	198.18
50% MED	36.7	90%	143.9
25% Q1	15.5	10%	8.1
0% MIN	1	5%	4.8
		1%	2.2
RANGE	277		
Q3-Q1	70.3		
MODE	24.6		

EXTREMES

LOWEST	HIGHEST
1	274
1.3	274
1.5	274
1.6	278
1.6	278

MISSING VALUE	.
COUNT	1
% COUNT/NOBS	0.15

UNIVARIATE

VARIABLE=BTNKH

MOMENTS

N	651	SUM WGTS	651
MEAN	4.50768	SUM	2934.5
STD DEV	6.37402	VARIANCE	40.6282
SKEWNESS	2.92123	KURTOSIS	10.8289
USS	39636.1	CSS	26408.3
CV	141.404	STD MEAN	0.249818
T:MEAN=0	18.0439	PROB>|T|	0.0001
SGN RANK	103523	PROB>|S|	0.0001
NUM ¬= 0	643		

QUANTILES(DEF=4)

100% MAX	48.9	99%	34.336
75% Q3	5.6	95%	18.46
50% MED	2.1	90%	11.78
25% Q1	0.8	10%	0.3
0% MIN	0	5%	0.1
		1%	0
RANGE	48.9		
Q3-Q1	4.8		
MODE	0.1		

EXTREMES

LOWEST	HIGHEST
0	36.2
0	36.3
0	39.5
0	41.2
0	48.9

MISSING VALUE	.
COUNT	1
% COUNT/NOBS	0.15

UNIVARIATE

VARIABLE=BTNKK

MOMENTS

N	651	SUM WGTS	651
MEAN	2.4533	SUM	1597.1
STD DEV	3.73902	VARIANCE	13.9802
SKEWNESS	3.27652	KURTOSIS	14.9356
USS	13005.3	CSS	9087.16
CV	152.407	STD MEAN	0.146544
T:MEAN=0	16.7411	PROB>|T|	0.0001
SGN RANK	98439	PROB>|S|	0.0001
NUM ¬= 0	627		

QUANTILES(DEF=4)

100% MAX	33.3	99%	18.272
75% Q3	2.9	95%	10.2
50% MED	1.1	90%	7.1
25% Q1	0.4	10%	0.1
0% MIN	0	5%	0.1
		1%	0
RANGE	33.3		
Q3-Q1	2.5		
MODE	0.1		

EXTREMES

LOWEST	HIGHEST
0	20.9
0	21.8
0	24.7
0	26.7
0	33.3

MISSING VALUE	.
COUNT	1
% COUNT/NOBS	0.15

UNIVARIATE

VARIABLE=TOBLCW

MOMENTS

N	651	SUM WGTS	651
MEAN	57.9101	SUM	37699.5
STD DEV	57.4594	VARIANCE	3301.58
SKEWNESS	2.65554	KURTOSIS	10.0014
USS	4329208	CSS	2146025
CV	99.2216	STD MEAN	2.25201
T:MEAN=0	25.7149	PROB>|T|	0.0001
SGN RANK	106113	PROB>|S|	0.0001
NUM ¬= 0	651		

QUANTILES(DEF=4)

100% MAX	435.9	99%	263.648
75% Q3	73.7	95%	169.74
50% MED	39	90%	119.22
25% Q1	21.3	10%	12.72
0% MIN	4	5%	9.7
		1%	5.116
RANGE	431.9		
Q3-Q1	52.4		
MODE	18.7		

EXTREMES

LOWEST	HIGHEST
4	275.1
4	299.2
4.1	422
4.6	433
4.6	435.9

MISSING VALUE .
COUNT 1
% COUNT/NOBS 0.15

UNIVARIATE

VARIABLE=LOBLCW

MOMENTS

N	651	SUM WGTS	651
MEAN	3.76651	SUM	2452
STD DEV	4.25095	VARIANCE	18.0705
SKEWNESS	2.74596	KURTOSIS	10.0592
USS	20981.3	CSS	11745.8
CV	112.862	STD MEAN	0.166608
T:MEAN=0	22.6071	PROB>|T|	0.0001
SGN RANK	105138	PROB>|S|	0.0001
NUM ¬= 0	648		

QUANTILES(DEF=4)

100% MAX	30.3	99%	23.468
75% Q3	4.9	95%	11.84
50% MED	2.4	90%	8.38
25% Q1	1.1	10%	0.5
0% MIN	0	5%	0.3
		1%	0.1
RANGE	30.3		
Q3-Q1	3.8		
MODE	1.1		

EXTREMES

LOWEST	HIGHEST
0	25.2
0	26.3
0	28.6
0.1	28.8
0.1	30.3

MISSING VALUE .
COUNT 1
% COUNT/NOBS 0.15

UNIVARIATE

VARIABLE=INITRA

MOMENTS

N	651	SUM WGTS	651
MEAN	4.14077	SUM	2695.64
STD DEV	3.72637	VARIANCE	13.8858
SKEWNESS	2.25138	KURTOSIS	8.85613
USS	20187.8	CSS	9025.79
CV	89.9922	STD MEAN	0.146048
T:MEAN=0	28.3521	PROB>|T|	0.0001
SGN RANK	106113	PROB>|S|	0.0001
NUM ¬= 0	651		

QUANTILES(DEF=4)

100% MAX	31.65	99%	17.4344
75% Q3	5.52	95%	11.518
50% MED	3.05	90%	9.13
25% Q1	1.57	10%	0.752
0% MIN	0.17	5%	0.506
		1%	0.2304
RANGE	31.48		
Q3-Q1	3.95		
MODE	0.72		

EXTREMES

LOWEST	HIGHEST
0.17	18.95
0.19	19.14
0.2	20.83
0.22	29.37
0.22	31.65

MISSING VALUE .
COUNT 1
% COUNT/NOBS 0.15

UNIVARIATE

VARIABLE=FINARA

MOMENTS

N	651	SUM WGTS	651
MEAN	4.43668	SUM	2888.28
STD DEV	4.44017	VARIANCE	19.7151
SKEWNESS	2.77366	KURTOSIS	13.8298
USS	25629.2	CSS	12814.8
CV	100.079	STD MEAN	0.174024
T:MEAN=0	25.4947	PROB>|T|	0.0001
SGN RANK	106113	PROB>|S|	0.0001
NUM ¬= 0	651		

QUANTILES(DEF=4)

100% MAX	41.5	99%	20.624
75% Q3	5.85	95%	13.054
50% MED	3	90%	10.24
25% Q1	1.55	10%	0.712
0% MIN	0.17	5%	0.482
		1%	0.21
RANGE	41.33		
Q3-Q1	4.3		
MODE	1.47		

EXTREMES

LOWEST	HIGHEST
0.17	21.11
0.19	22.04
0.19	27.05
0.19	37.91
0.2	41.5

MISSING VALUE .
COUNT 1
% COUNT/NOBS 0.15

APPENDIX B

CHI-SQUARE TEST FOR DIFFERENCE

TANK LOSS FRACTION			
BLUE UNITS WITH MAIN BATTLE TANK	BLUE UNITS WITHOUT MAIN BATTLE TANK	RED UNITS WITH MAIN BATTLE TANK	RED UNITS WITHOUT MAIN BATTLE TANK
.376	.208	.166	.160
.149	.244	.111	.145
.325	.358	.144	.065
.233	.352	.179	.184
.374	.274	.132	.042
.298	.258	.203	.158
.168	.309	.160	.031
.289	.254	.141	.173
.134	.314	.217	.051
.236	.392	.183	.137
.324	.106	.058	.132
.318	.261	.025	.052
.392	.349	.092	.034
.372	.197	.194	.063
.155	.261	.105	.109
	.349	.217	.075
	.197	.141	.049
	.261	.152	.097
	.261	.126	.118
	.275		
	.423		
	.151		
	.169		
	.136		
	.186		
	.117		

The following equation was used:

$$\chi^2 = \Sigma\frac{(f_{ij} - e_{ij})^2}{e_{ij}}$$

f - Actual frequency
e - Expected frequency

$$e_{ij} = \frac{R_i C_j}{T} \qquad e_{ij} \geq 5$$

R_i - Row total
C_j - Column total
T - Grand total

The level of significance--α=.05.

TANK LOSS FRACTION OF BLUE UNITS	BLUE UNITS WITH MAIN BATTLE TANK	BLUE UNITS WITHOUT MAIN BATTLE TANK	FREQUENCY ROW SUM
0 - .25	5	9	14
.25 - ∞	10	4	24
COLUMN SUM	15	23	

5.5	8.5	EXPECTED FREQUENCY
9.5	14.5	

H_0: There is no difference between the vulnerability of the blue units

H_1: There is a difference

χ^2 = .117
df=1

$\chi^2_{.95}$ = 3.841
df=1

Cannot reject H_0
No significant difference

TANK LOSS FRACTION OF RED UNITS	BLUE UNITS WITH MAIN BATTLE TANK	BLUE UNITS WITHOUT MAIN BATTLE TANK	FREQUENCY ROW SUM
0 - .1	9	4	13
.1 - ∞	6	19	25
COLUMN SUM	15	23	

5.13	7.87	EXPECTED FREQUENCY
9.87	15.13	

H_0: There is no difference between the lethality of the blue units

H_1: There is a difference

$\chi^2 = 7.3$ df=1 $\chi^2_{.95} = 3.841$ Reject H_0 so there is a significant difference

TANK LOSS FRACTION OF RED UNITS	RED UNITS WITH MAIN BATTLE TANK	RED UNITS WITHOUT MAIN BATTLE TANK	FREQUENCY ROW SUM
0 - .13	6	12	18
.13 - ∞	13	7	20
COLUMN SUM	19	19	

9	9	EXPECTED FREQUENCY
10	10	

H_0: There is no difference between the vulnerability of the red units

H_1: There is a difference

$\chi^2 = 3.8$ df=1 $\chi^2_{.95} = 3.841$ Cannot reject H_0 No significant difference

TANK LOSS FRACTION OF BLUE UNITS	RED UNITS WITH MAIN BATTLE TANK	RED UNITS WITHOUT MAIN BATTLE TANK	FREQUENCY ROW SUM
0 - .3	11	13	24
.3 - ∞	8	6	14
COLUMN SUM	19	19	

12	12	EXPECTED FREQUENCY
7	7	

H_0: There is no difference between the lethality of the blue units

H_1: There is a difference

$\chi^2 = .4527.3$ df=1 $\qquad \chi^2_{.95} = 3.841$ $\qquad$ Cannot reject H_0 No significant difference

LIST OF REFERENCES

1. Hartman, James K., "AGGREGATION IN COMBAT MODELS," Lecture notes in AGGREGATED COMBAT MODELLING presented at the Naval Postgraduate School, Monterey, CA, 1985.

2. Metzger, J. J., "FORCE EVALUATION MODEL," paper U.S. Army Concepts Analysis Agency, Bethesda, MD.

3. Hartman, James K., "THE ATCAL ATTRITION MODEL," Lecture notes in AGGREGATED COMBAT MODELLING presented at the Naval Postgraduate School, Monterey, CA, 1985.

4. Telephone conversation between Maj. Cyrus Staniec, FOT, U.S. Army CAA and the author, 13 May 1989.

5. U.S. Army CAA Bethesda, MD, memorandum, Subject: FORCEM ATCAL ANALYSIS CONTINUATION, 31 October 1988.

6. Chambers, J. M., and others, "Graphical Methods for Data Analysis," Wadsworth International Group, 1983.

7. Conover, W. J., "Practical Nonparametric Statistics," 2d ed., John Wiley and Sons, 1980.

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Thesis
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c.1 Graphical analysis of the sensitivites of ATCAL in the FORCEM model.